Thermal Masses and Equilibrium Rates in the Quark Gluon Phase

Jan-e Alam[a], Pradip Roy[a], Sourav Sarkar[a], Sibaji Raha[b] and Bikash Sinha[a,c]

a) *Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Calcutta 700 064, India*

b) *Bose Institute, 93/1, A. P. C. Road, Calcutta 700 009, India*

c) *Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Calcutta 700 064, India*

Abstract

We apply the momentum integrated Boltzmann transport equation to study the time evolution of various quark flavours in the central region of ultra-relativistic heavy ion collisions. The effects of thermal masses for quarks and gluons are incorporated to take into account the in-medium properties of these ingredients of the putative quark gluon plasma. We find that even under very optimistic conditions, complete chemical equilibration in the quark gluon plasma appears unlikely.

Quantum Chromodynamics (QCD), the theory of strong interactions, predicts that at very high temperature and/or density, the colourless hadronic matter dissolves into its coloured constituents, the quarks and gluons, such that the bulk properties of strongly interacting matter are governed by these degrees of freedom. Such a locally colour-deconfined phase of matter is known as Quark Gluon Plasma (QGP). It is expected that the temperature and density achievable in ultrarelativistic collisions of heavy ions is favourable for QGP formation, although transiently.

Many aspects of this transition, *e.g.*, the order of the phase transition, the value of the critical temperature, signals of the transition, thermodynamic equilibration , are still uncertain. So far, most of the works assume thermodynamic equilibrium throughout the evolution history, after an initial proper time τ_i (~ 1 fm/c). Thus the thermodynamic quantities like pressure, entropy and temperature have direct meaning with respect to an equilibrated system. It has however been recently shown that the approach to equilibrium in ultrarelativistic heavy ion collisions proceeds through a succession of many time scales[1,2]. In particular, the dominance of the *gg* cross section over the *qg* or *qq* cross sections[3] was argued to imply that the gluons equilibrate among themsleves appreciably earlier than the whole system of quarks and gluons. In a recent work[1] , we showed that the time scale of *kinetic* equilibration for the light quarks (u, d) is ~ 1 fm/c; for massive quarks, it increases with the mass to rather large values. The significance of these considerations for QGP diagnostics has been discussed in references 4 and 5; the emission of particles from the pre-equilibrium era ($\tau_g \leq \tau \leq \tau_{th}$), τ_{th} denoting the time for full thermodynamic equilibrium, may indeed populate the invariant mass or p_T windows thought to be relevant for signals from the thermalised QGP.

In this letter, we study the *chemical* evolution of the quarks in the system from the epoch τ_g (the proper time when gluons thermalise) onward. Chemical equilibration has very important implications for the signal of QGP formation[4,5,6] . It would be ideal to study this problem in the QCD based kinetic theory and attempts along these lines have been made recently[7] . Such calculations, appealing

as they are, rely heavily on rather involved numerical simulations and also suffer from dependence on parameters needed to mimic the non-perturbative effects. We propose, as in ref. 1 , a scenario which retains the essence of the kinetic theory to the following extent : the gluons thermalise *completely* at a proper time τ_g earlier than the quarks. The gluons carry about half of the total momentum and the quarks carry a tiny fraction. We restrict ourselves to the central rapidity region where the number of valence quarks is assumed to be negligibly small. We shall, however, relax this condition when we look at the situation at SPS energies.

In order to treat the equilibration of the species one must follow the microscopic evolution of the phase space distribution function $f(x^\mu, p^\mu)$, governed by the Boltzmann transport equation :

$$\hat{L}\{f_q\} = \hat{C}\{f_q\} \tag{1}$$

where $\hat{L}$ is the Liouville operator and $\hat{C}$ the collision operator, the subscript q denoting a quark flavour. The Boltzmann equation is a non-linear, integro-differential equation for the phase space distribution function of the particles. For our present purpose, however, the problem can be addressed in a rather simple manner without losing much information on the time scales for chemical equilibrium.

The decoupling of the relic particles in the early universe was studied[8,9] by integrating the Boltzmann equation over the momentum of the particle to obtain an equation for the evolution of number density. We use a similar approach to look at the number density evolution of the partons in ultrarelativistic collisions of heavy ions; the major difference between the two cases is that the expansion dynamics in the early universe (*big bang*) is governed by the Hubble expansion[9] whereas in the case of heavy ion collision (*mini bang*), the expansion dynamics is assumed to be governed by the boost invariant (Bjorken) solution of relativistic hydrodynamics[10] . The equation for the number density evolution then reads

$$\frac{dn_q}{d\tau} + \frac{n_q}{\tau} = \frac{g_q}{(2\pi)^3} \int \{\hat{C}\} \frac{d^3p}{E} \tag{2}$$

where n_q is non-equilibrium quark density, τ the proper time and g_q the statistical degeneracy of the quarks of flavour q.

Clearly, the time evolution of n_q would be largely governed by the various processes determining the collision term in eq. (2). The most important contribution to this is from gluon fusion from $(gg \rightarrow q\bar{q})$.

Since we have considered complete equilibration for gluons, reactions like $gg \leftrightarrow ggg$ need not be included[1] ; the gluon number density at each instant is uniquely determined by the temperature of the system. However the gluons in a finite temperature system acquire a sizable thermal mass may qualify as real excitations[11,12] . Thus the gluon decay process $(g \rightarrow q\bar{q})$ and its inverse is expected to play a major role; in particular it has been argued in the literature[11] that for small quark masses $(m_q^{eff}/T << 1)$, the gluon decay dominates over the fusion process. We must therefore take the gluon decay into account while evaluating the collision term. We ignore the quantum effects as the temperature is expected to be very high but include relativistic effects for the same reason. Thus gluon are described by the relativistic Maxwell-Boltzman statisics, $f_g(E) = g_g/(2\pi)^3 e^{-E/T}$. Then eq.(2) can be written as:

$$\frac{dn_q}{d\tau} = -\frac{n_q}{\tau} - (R_{gg\leftrightarrow q\bar{q}}(T) + R_{g\leftrightarrow q\bar{q}}(T))\frac{(n_q^2 - n_{eq}^2)}{n_{eq}^2} \tag{3}$$

where $n_{eq}(T) = g_q/(2\pi)^3 \int d^3p/(e^{E/T} + 1)$, is the equilibrium density. Eq.(3) is a particular form of the Riccati equation; there is no general closed form solution of this equation.

In deriving eq.(3) we have assumed T (or CP) invariance, i.e., $|M|_{gg\rightarrow q\bar{q}} = |M|_{q\bar{q}\rightarrow gg}$ and $|M|_{g\rightarrow q\bar{q}} = |M|_{q\bar{q}\rightarrow g}$. $R_{gg\rightarrow q\bar{q}}(T)$ is the quark production rate per unit four volume by the reaction $gg \rightarrow q\bar{q}$ and $R_{g\rightarrow q\bar{q}}$ gives the same for the gluon decay[11,12], $g \rightarrow q\bar{q}$. The thermal quark masses have been taken into account using $m_q^{eff}(T) = \sqrt{m_{current}^2 + m_{th}^2}$ in all the rates. In the present context in a chemically non-equilibrated scenario m_{th} is given by[13]

$$m_{th}^2 = (1 + \frac{r_q}{2})\frac{g^2 T^2}{9} \tag{4}$$

where g is the colour charge, $r_q \equiv n_q/n_{eq}$. The full derivation of eq.(3) has been omitted here for the sake of brevity; it will be reported elsewhere[14] . Eq.(3) is a very convenient form of the Boltzmann transport equation with all the relevant

features for our present purpose. The first term on the right hand side of the equation represents the "dilution" of the density due to one dimensional expansion, the destruction of the quarks is proportional to the annihilation rates of $q\bar{q}$ and destruction is balanced by creation through the inverse reaction when $n_q = n_{eq}$. The last term represents the change in quark density due to gluon decay and its inverse process, arising from the thermal gluon mass. Note that there is no overcounting as the phase space integrations involved in the gluon fusion and the gluon decay channels are over different domains. For a detailed discussion see ref. 11.

To solve eq.(3) for $n_q(\tau)$ we need to know $T(\tau)$, which is obtained by solving the hydrodynamic equation

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = 0 \tag{5}$$

where P is the pressure and ϵ the energy density. An equation of state relating P and ϵ can be parametrized in the form

$$P = c_s^2 \epsilon \tag{6}$$

Then, for the non-equilibrium configuration, we can write

$$\epsilon = [a r_g + b r_q(\tau)] T^4 \tag{7}$$

where $a = (8\pi^2)/45 \, c_s^{-2}$ and $b = (7\pi^2)/60 \, N_f \, c_s^{-2}$. The medium effects enter eq. (7) through the thermal mass of the quarks; the higher the temperature, the higher the thermal mass. Therefore the equation of state may be parametrized as $P = c_s^2(T)\epsilon$, where $c_s^{-2} = \left[\frac{T}{g_{eff}(T)} \frac{dg_{eff}}{dT} + 3\right]$ with $g_{eff}(T) = (\epsilon + P)/(2\pi^2 T^4)/45$. This would amount to making a and b in eq. (7) functions of T. For the cases considered here, such effects are negligibly small.

In chemical equilibrium, r_q for all species should be unity and in correspondence with our premise, we take $r_g = 1$ all through. N_f is the *effective* number of *massless* flavours; appearance of quark masses amounts to reducing the actual number of flavours[15] . For the present, we confine our attention only to the three lightest flavours, u, d and s. The difference between the $u(d)$ quarks and s quarks arises only in terms of their current mass. We have verified that because of the high initial

temperature and the corresponding large thermal masses, this amounts to a very small change in the effective N_f.

Equations (5) and (7) together yield,

$$T^3\tau = \left[\frac{a + br_q(\tau_g)}{a + br_q(\tau)}\right]^{3/4} T_g^3\tau_g \qquad (8)$$

which is the cooling law in the present non-equilibrium scenario.

To solve eq.(3), we need to specify the initial conditions. There is no unique pre-scription for evaluating them at the present state of the art. Biró et al. estimated[16] the initial parton density from the HIJING algorithm[17] . On the other hand, Shuryak and Xiong[5], advocated, as in ref. 1, the use of "perturbative" QCD(pQCD), i.e., the deep inelastic structure functions for the estimation of the initial quark density. Given the importance of the issue, we have, in the present work, used both these estimates.

In the pQCD approach, the initial values of $n_q(\tau_g)$'s are obtained from the integration of the structure function[18],

$$n_q = 2A \times \frac{\int_{x_{min}}^{1} u_q(x)dx}{\pi R_A^2\tau_g} \qquad (9)$$

where A is the mass number of the colliding nuclei (208 in our case for Pb), x_{min} is the minimum value of x, taken[1,5] to be 0.02, dictated by the applicability of perturbative QCD. It is well known that the structure functions depend on x and Q^2 and therefore, it is important to choose the proper range of Q^2. The range of Q^2 appropriate for our present context can be determined by comparing the values of the running coupling constant $\alpha_s(Q^2)$ with $\alpha_s(T = T_g)$[19] . Thus for initial temperatures T_g of 500 and 660 MeV (corresponding to $\tau_g = 0.3$ and 0.25 fm/c) at RHIC and LHC energies[2], respectively, the appropriate ranges of Q^2 are 25 and 55 GeV^2. Then, the values of $n_q(\tau_g)$ estimated from eq. (9) come out to be 1.26 and 1.33 fm^{-3} at RHIC and LHC energies. From ref. 16, these values are 0.7 and 2.8 fm^{-3}, respectively.

Eqs.(3), (4) and (8) were solved self consistently by the Runge -Kutta method to obtain the non-equilibrium density $n_q(\tau)$. As is evident from the above discussions,

the non-equilibrium density n_q has an explicit dependence on τ and an implicit dependence on τ through $T(\tau)$. But the equilibrium density $n_{eq}(T)$ has only an implicit dependence on τ through $T(\tau)$. The ratio r_q thus assumes an *universal* feature, since the implicit time dependence gets eliminated. The time dependence of the ratio r_q can then be used as a ready marker for chemical equilibrium; the time at which the explicit time dependence of r_q vanishes, *simultaneously with $r_q \rightarrow$ 1*, corresponds to the time for chemical equilibration for the flavour q.

In contrast to the earlier work of Biró *et al*[16] , we have not included reactions like $q\bar{q} \rightarrow Q\bar{Q}$, since the initial system is dominated by gluons[2,7]; the quark density is very low compared to the gluons. These reactions are suppressed by a factor of about 1/16 compared to the case when the quarks are in complete equilibrium[5]. Furthermore, we would like to emphasize at this juncture that we are looking at an optimistic scenario, where the gluons have completely equilibrated already.

The relative importance of the gluon fusion ($gg \rightarrow q\bar{q}$) and gluon decay ($g \rightarrow q\bar{q}$) channels is obviously a crucial issue in the present context. In fig. 1, we show the reaction rates for these two channels. If one considers the thermal contribution to the quark masses also, then the effective quark mass becomes $m_q^{eff}(T) = \sqrt{m_{current}^2 + m_{th}^2}$. One can readily see from fig. 1 that the gluon fusion dominates over the gluon decay modes for the entire range of interest; it has however to be noticed that the higher the m_q^{eff} the greater the difference between them, implying that for light quarks(u,d and even s), the decay channel does play an important role.

The ratios $r_q = n_q/n_{eq}$ are plotted in fig.2(a) for RHIC energies for the initial conditions specified by eq. (9). At these energies, the ratios for $u(d)$ quarks and s quarks are the same at early times but at late times the $u(d)$ quarks dominate over the s quarks. The reason is that at early times the thermal mass ($\propto T$) dominates over the current quark mass for all flavours. At later times, however, the thermal mass decreases with decreasing temperature and the difference in the current masses of $u(d)$ ($\sim 10 MeV$) and s ($\sim 150 MeV$) quarks becomes important. There occurs thus some Boltzmann suppression for s quarks relative to $u(d)$ quarks.

The same calculation has also been carried out for the initial condition specified by the algorithm HIJING. At RHIC, the initial r_q for all three flavours are lower than those estimated from eq.(9) (fig. 2(a)). As is to be expected, this makes equilibrium even less likely, which is borne out in our detailed calculation. For LHC, the initial r_q's are systematically higher than those obtained from eq.(9); one could thus naively expect that equilibration may become easier. We therefore show in fig. 2 (b) the result for this case also. One sees no sign of saturation with τ for either u(d) or s quarks.

In fact, one can also estimate the initial quark density from the prescription $N_q = \sigma_q \cdot T_{AA}(0)$ where $T_{AA}(0)$ is the overlap function for central AA collision and σ_q the production cross section for the quark[20] q , one would obtain significantly lower values of $n_q(\tau_g)$, or equivalently $r_q(\tau_g)$. (e.g. $n_q(\tau_g) \sim 0.48$ at RHIC compared to 1.26 obtained from eq.(9)). This would then imply lesser probability for chemical equilibrium. We have verified that this is indeed so; the qualitative behaviour of the results shown above remain unaltered while quantitatively the conclusions are strengthened at all the energy domains considered here.

We have so far considered futuristic scenarios at RHIC and LHC where the assumption of a baryon free region is perhaps applicable. For the presnt, namely the SPS energies, such an approximation is not at all appropriate; it is expected that there would be a substantial amount of stopping at these energies[21]. Nevertheless, study of flavour equilibration at SPS energies becomes a very pertinent and timely issue. We have therefore investigated two extreme scenarios– one of total transparency as above and one of complete stopping.

In fig. 3 we show the time evolution of r_q and T as a function of τ for the transparent case. The initial conditions are determined by eq.(9) alone, as, to our knowledge, the corresponding estimates from HIJING do not exist in the literature. We find that at early times r_s is slightly larger than r_u or r_d; this is due to the initial normalisation. The non-equilibrium density $n_{u(d)} \sim n_s$ but the equilibrium density for s quarks is less than that of u or d quarks because of the difference in their

masses. This results in r_s being larger than $r_{u(d)}$. Expectedly, this effect washes out with progress of time. In any case, there is no hint of flavour equilibration at SPS energies either, under the assumption of complete transparency. In the other extreme case, that of complete stopping , however, the situation is most interesting. Because of the presence of a large number of valence u and d quarks, the initial values of r_u and r_d are very large. For central collisions of lead nuclei, the initial value of $r_{u(d)}$ turns out to be 0.98(1.06), taking into account both valence and sea contributions. As τ increases, r_q tends to 1, r_u from below and r_d from above. Nevertheless, the stationarity of r_q with τ does not seem to be achievable within the life time of the QGP. It is thus fair to conclude that at SPS, u and d quarks, although not in equilibrium, may not be too far from equilibrium in the event of complete stopping. The situation for the s quark is, however, quite different as all the initial s quarks come from the sea. Thus $r_s(\tau_g)$ is still rather small. But now, one has to include $u\bar{u}(d\bar{d}) \rightarrow s\bar{s}$, in addition to gluon fusion and decay channels. We observe that even for such a favourable situation the time evolution of r_s is indistinguishable from that shown in fig. 3.

These findings are obviously in contrast with the earlier expectations of other authors[22,23] where s quarks were expected to come to equilibrium at SPS energies. The difference is ascribable to the effect of the thermal masses, which must be included if proper account of the in-medium effects is to be taken. We have shown here that these effects do play a very important role indeed and may affect the conclusions nontrivially. Obviously, implications of these issues in QGP diagnostics is an urgent task; work along these lines is in progress.

References

1. J. Alam, S. Raha and B. Sinha, Phys. Rev. Lett. **73** (1994) 1895

2. E. Shuryak, Phys. Rev. Lett. **68** (1992) 3270

3. R. Cutler and D. Sivers, Phys. Rev. **D17** (1978) 196

4. S. Raha, Phys. Scr. **T32** (1990) 180 ; S. Chakrabarty, S. Raha and B. Sinha, Mod. Phys. Lett. **A7** (1992) 927

5. E. Shuryak and L. Xiong, Phys. Rev. Lett. **70** (1993) 2241

6. K. Geiger and J. Kapusta, Phys. Rev. Lett. **70**, (1993) 1920; Phys. Rev. **D47** (1993) 4905

7. K. Geiger, Phys. Rep. **258** (1995) 237

8. E. W. Kolb and S. Wolfram, Nucl. Phys. **B172** (1980) 224

9. E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley, New York), 1990.

10. J. D. Bjorken, Phys. Rev. **D27** (1993) 140

11. T. Altherr and D. Seibert, Phys. Rev. **C49** (1994) 1684

12. T. Altherr and D. Seibert, Phys. Lett. **B 313** (1993) 149

13. C. T. Traxler and M. H. Thoma, Phys. Rev. **C53** (1996)1348.

14. J. Alam *et al*, to be published.

15. R. C. Hwa and K. Kajantie, Phys. Rev. **D32** (1985) 1109

16. T. S. Bíró, E. van Doorn, B. Müller, M. H. Thoma and X. N. Wang, Phys. Rev. **C48** (1993) 1275

17. X. N. Wang, Phys. Rev. **D43** (1991) 104; X. N. Wang and M. Gyulassy, Phys. Rev. **D44** (1992) 3501

18. M. Glück, E. Reya and A. Vogt, Z. Phys. **C48** (1990) 471

19. F. Karsch, Z. Phys. **C38** (1988) 147

20. B. Müller and X. N. Wang, Phys. Rev. Lett. **68** (1992) 2437

21. F. Plasil in Physics and Astrophysics of Quark-Gluon Plasma, eds. B. Sinha, Y. P. Viyogi and S. Raha, (World Scientific, Singapore, 1994).

22. T. Matsui, B. Svetitsky and L. D. Mclerran, Phys. Rev. **D34**, (1986) 783.

23. J. Rafelski, J. Letessier and A. Tounsi, in Strangeness in Hadronic Matter, edited by J. Rafelski, American Institute of Physics, NY, 1995.

Figure Captions

Fig. 1 : Comparison of quark production rates due to gluon fusion and gluon decay, the value of α_s is calculated from the parametrisation of ref 19.

Fig. 2 : Ratio of non-equilibrium density to equilibrium density and temperature as functions of time (a) at RHIC energy for initial condition given by eq.(9) and (b) at LHC energy for intial condition given by eq.(9) and ref. 16.

Fig. 3 : Same as fig. 2 (a), at SPS energy (see text).

Running title:

Thermal Masses and Equilibrium Rates .....